CSCI3390-Assignment 1.

due September 6, 2018

You will need the material in Lecture 1 to complete this assignment.

1   Encoding problem instances as strings.

The scheme used for encoding graphs is illustrated in Figure 1. I have deliberately chosen a flawed encoding scheme so that you can correct it in the last part of this problem, but for all other parts of the problem, you should stick with this method.

A *Hamiltonian path* in a graph is a path that includes every vertex of the graph exactly once. A *Hamiltonian circuit* is a path that starts and ends at the same vertex and visits every other vertex exactly once.

(a) The pictured graph has a Hamiltonian circuit. Find it.

(b) Eliminate one edge of the pictured graph so that the resulting graph has a Hamiltonian path but not a Hamiltonian circuit. (Tell which edge you are elminating, and what the path is.)

(c) Let $HAMPATH$ be the language representing the decision problem 'Does this graph have a Hamiltonian path?' relative to this encoding. Let $HAMCIRC$ be the language representing the problem 'Does this graph have a Hamiltonian cricut?' Let w be the following string.

$$1\#2\#1\#3\#1\#4\#2\#4.$$

Is $w \in$ HAMPATH? Is $w \in$ HAMCIRCUIT? (Don't just tell me yes or no; explain your answer for all parts.) It helps to draw a picture.
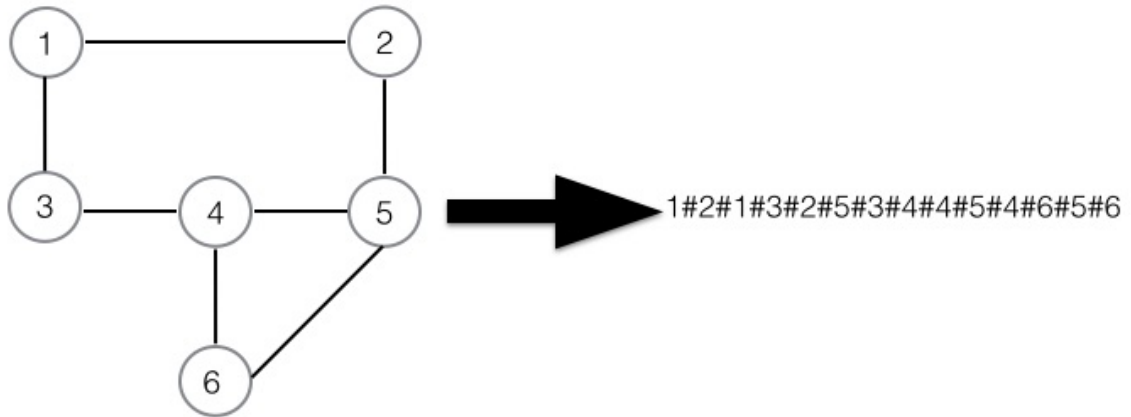
Figure 1: The graph encoding scheme for Problem 1.

(d) We can encode anything over the two-letter alphabet $\{0, 1\}$. Describe how to adapt the encoding scheme for graphs so that it uses only these two symbols. (A simple answer is: 'encode the vertex numbers in binary, rather than decimal' but that doesn't tell us how to handle the separator symbol # ! There are, however, several different correct answers.)

(e) Let V and E represent, respectively, the sets of vertices and edges of a graph, and $|V|$ and $|E|$ the number of elements in these sets. Write an expression using these numbers that gives the length of the encoding of the graph, or at least a reasonable upper bound for the length of the encoding. (This problem is a little subtle: It is tempting to write $4 \cdot |E|$ because we use about 4 symbols to encode each edge, or $4|V|^2$, because there are $|V|^2$ pairs of vertices. But these are not correct, because we will need more than one symbol to encode a vertex number if we have ten or more vertices.)

(f) The problem with this encoding scheme is that it only includes vertices that are endpoints of edges. But a general graph can have *isolated* vertices that are not connected by an edge to any other vertex. Describe a scheme for encoding graphs by strings that behaves correctly for all undirected graphs. (There are many good answers here.) Give an example illustrating how your encoding works.
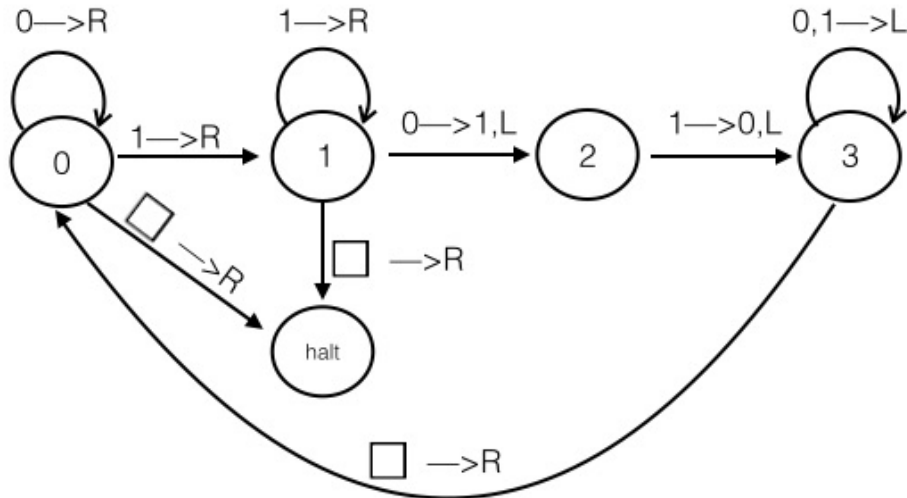
Figure 2: The Turing machine for Problem 2.

2 A Turing Machine

This problem refers to the Turing machine in Figure 2.

(a) Trace the execution of the machine (that is, give the complete sequence of configurations) on the input 000, 11 and 1010. You can do this by hand (which is a little tedious for the last example, but not too terrible), or you can use the Turing machine simulator program. In the latter case, you should include your specification file with your submitted assignment, as well as the output printed by the program.

(b) Describe *in general* the function f computed by this Turing machine. When you figure out the answer, you should be able to find $f(0110110110110110110110110)$ quickly, without running the machine.

(c) Here is a harder, but important question. How many steps does it take (*i.e,* how long is the complete sequence of configurations) on an input of length n? The answer depends on the input string, of course: If it is 0^n (all zeros) then the computation terminates very quickly. Here you should give the *worst-case* answer—what input causes the computation to take as long as possible, and how many steps does it require?