1) In the following situations, indicate whether \( f = O(g) \) or \( f = \Omega(g) \), or \( f = \Theta(g) \). Prove your result.

(a) \( f(n) = n - 100 \), \( g(n) = 2n - 200 \).
(b) \( f(n) = 100n + \log n \), \( g(n) = n + (\log n)^2 \).
(c) \( f(n) = n \log n \), \( g(n) = 10n \log 10n \).
(d) \( f(n) = 10 \log n \), \( g(n) = \log(n^2) \).
(e) \( f(n) = n^{1.01} \), \( g(n) = n \log^2 n \).
(f) \( f(n) = n^2 / \log n \), \( g(n) = n(\log n)^2 \).
(g) \( f(n) = \sqrt{n} \), \( g(n) = (\log n)^3 \).
(h) \( f(n) = 2^n \), \( g(n) = 3^n \).
(i) \( f(n) = (\log n)^{\log n} \), \( g(n) = 2^{(\log n)^2} \).

2) Show that, if \( c \) is a positive real number, than \( g(n) = 1 + c + c^2 + ... + c^n \) is:

(a) \( \Theta(1) \) if \( c < 1 \).
(b) \( \Theta(n) \) if \( c = 1 \).
(c) \( \Theta(c^n) \) if \( c > 1 \).

3) Use induction method to show that Fibonacci number \( F_n \) grows exponentially and to find some bound.

(a) Prove that \( F_n \geq 2^{0.5n} \) for \( n \geq 6 \).
(b) Find a constant \( c < 1 \) such that \( F_n \leq 2^{cn} \) for all \( n \geq 0 \). Prove your result.
(c) What is the largest number \( c \) you can find for which \( F_n = \Omega(2^{cn}) \).

4) Consider a \( 2^n \times 2^n \) chess board with one arbitrary tile removed.

(a) Using induction method, prove that any such board can be tiled without gaps using L-shaped pieces, each composed of 3 squares.
(b) Write a recursive program to print out the tiling.