1. Consider the propositional formula 

\[(p \land q) \rightarrow \neg r \rightarrow (q \rightarrow r)\].

(a) Find all settings of truth values for \(p\), \(q\) and \(r\) for which this formula has the value true.
(b) Write an equivalent formula using only the connectives \(\land\) and \(\neg\). Make the formula as simple as possible.

2. (a) Consider the formula

\[\exists x (x < 1 \land \forall y (y < 1 \rightarrow y \leq x))\].

Tell whether this formula is true assuming that the universe of discourse is
(i) the set \(\mathbb{Z}\) of integers; (ii) the set \(\mathbb{Z}^+\) of positive integers; (iii) the set \(\mathbb{R}\) of real numbers. Explain briefly.
(b) Recall the definition of the gcd of two integers: \(d\) is the gcd of \(m\) and \(n\) if \(d\) divides \(m\) and \(n\), and all common divisors of \(m\) and \(n\) divide \(d\). Write a quantified formula (assuming the universe of discourse is \(\mathbb{Z}\)), using quantifiers, propositional connectives, and the symbol \(|\) for ‘divides’.

3. (a) if \(A\) and \(B\) are sets for which \(A \cup B = A\), what can you conclude about \(A\) and \(B\)?
(b) if \(A\) and \(B\) are sets for which \(A \cap B = A\), what can you conclude about \(A\) and \(B\)?
(c) Determine the cardinality of the power set \(|\mathcal{P}(\mathcal{P}(\emptyset))|\).

4. Find the necessary and sufficient conditions on \(x\) and \(y\) such that

\[\lfloor x \rfloor + \lceil y \rceil = \lceil x \rceil + \lfloor y \rfloor\]

and prove your answer is correct.

5. Consider the function \(f: \mathbb{N} \rightarrow \mathbb{Z}\) defined by

\[f(x) = \lfloor \frac{(-1)^x x}{2} \rfloor\].

Is \(f\) one-to-one? Is it onto? Explain.

6. Let \(f(n) = 3\sqrt{n}\) and \(g(n) = \sqrt{n}\). Compare the order of the two functions, is one the big \(O\) of the other?

7. Find \((435 \cdot 611) \mod 7\).