1 Q(4.4-8)

The coefficient is $C(17, 8) \cdot 3^8 \cdot 2^9$.

2 Q(4.4-14)

Let $m \leq \lceil n/2 \rceil$ and $m \geq 1$.

$$
\frac{C(n, m)}{C(n, m - 1)} = \frac{n!}{m!(n - m)!} \cdot \frac{(m - 1)!(n - m + 1)!}{n!} = \frac{(n - m + 1)}{m + 1}
$$

Since $m \leq \lfloor n/2 \rfloor$, we have $m \leq n/2$ and therefore $\frac{n - m + 1}{m} > 1$. We have $C(n, m) > C(n, m - 1)$ for $m \leq \lfloor n/2 \rfloor$ and $m \geq 1$. The completes the first half of the proof. The other half can be done similarly.

3 Q(4.4-22)

1. The left hand side is the number of ways to select $r$ objects from $n$ objects and then select $k$ objects from the chosen $r$ objects. Apparently, it equals to the number of ways to complete two tasks in a reverse order: choose $k$ objects from $n$ objects and then choose another $(n - k)$ objects from the rest to form a $r$-object group; the number is just the right hand side of the equation.

$$
\frac{n!}{(n-r)!} \frac{r!}{(r-k)!k!} = \frac{n!}{(n-r)!(r-k)!k!} \cdot \frac{n!}{(n-k)!k!} \frac{(n-k)!}{(r-k)!(n-r)!} = \frac{n!}{(n-r)!(r-k)!k!}
$$

2. We complete the proof.
4 Q(5.1-12)

The ways to form a 5-card poker hand is $C(52, 5)$. The ways to have exactly one ace in the 5 cards is $C(4, 1)C(52 - 4, 4)$. The probability is the ratio of them.

5 Q(5.1-14)

There are totally 13 kinds of cards. Each kind has 4 cards. To form a 5-card hand with 5 different kinds, we first choose 5 kinds with $C(13, 5)$ ways and then 1 card from each chosen kind with $C(4, 1)$ ways. Using the product rule, there are $C(13, 5)C(4, 1)$ ways to form the hand. The probability is $C(13, 5)C(4, 1)/C(52, 5)$.

6 Q(5.1-36)

To get a total of 8 by rolling two dices the outcomes can be 2 tuples (1,7), (2,6), (3,5), (4,4), (5,3), (6,2) and (7,1). The probability is 7/36.

To get a total of 8 by rolling three dices the outcomes are 3 tuples. If the first dice is 1, the other two needs to sum to 7, there are 6 ways. And if the first dice is 2, the other two needs to sum to 6, there are 5 ways. With the similar reasoning, we get all the ways to get a sum 8 using 3 dices. The number is 6+5+4+3+2+1=21. Therefore the probability is 21/216. It is much smaller than the probability of rolling two dices.

7 Q(5.1-40)

With 4 doors, if you do not change mind, the probability to hit the right door is apparently 1/4. If you choose another door after a door that does not hold the object is removed, there will be totally 8 ways to make a choice. There are only 3 ways to get the right door: you first select one of the three doors that do not hold the object and then you have 1 way to select the target at the second choice. The probability is 3/8. It is still bigger than the first strategy.

8 Q(5.2-6)

1. As discussed in the class, to get 1 precede 3 in the permutation, we select 2 positions from 3 and put 1 and 3 in there and keep the sequence. There are $C(3, 2)$ ways to implement this. To complete, we put 2 at the remaining position. Therefore the probability is $C(3, 2)/3! = 1/2$. Another way to
think about the problem is to realize that it is equal likely that 1 precedes 3 and 3 precedes 1. Therefore the probability is 1/2.

2. 1/2.

3. Apparently there are 2 ways to make 3 precede 1 and 2. The probability is 2/6=1/3.

9 Q(5.2-8)

1. \(C(n, 2)(n - 2)!/n! = 1/2\)
2. 1/2.
3. \((n - 1)(n - 2)!/n! = 1/n\)
4. \(C(n, 2)C(n - 2, 2)(n - 4)!/n! = 1/4\)
5. \(2C(n, 3)(n - 3)!/n! = 1/3\)

10 Q(5.2-18)

1. The birthdays in a week for two people have 7x7=49 possible outcomes. There are 7 outcomes for which they have the same birthday. Therefore the probability is 1/7.

2. If \(n > 7\), the probability of at least two people born in the same day of a week is 1. For \(n \leq 7\), we consider the chance that no two people were born in the same day. There are \(P(7, n)\) ways to assign different birthdays to the \(n\) person and therefore the probability is \(P(7, n)/7^n\). The probability that at least two people are born in the same day is \(1 - P(7, n)/7^n\).

3. If \(n = 1\), the chance is 0. If \(n = 2\), \(P\) is 1/7. If \(n = 3\), \(P\) is about 0.38. If \(n = 4\), \(P\) is about 0.65. We therefore need at 4 people to make the probability be greater than 1/2.

11 Q(5.2-30)

1. \(1/2^n\)
2. \(0.6^n\)
3. \(1/2 \cdot 1/2^2 \cdot \ldots \cdot 1/2^{10} = 1/2^{55}\)