Instructions: This exam is open book. You can use any reference books in the exam. But no computers are allowed. Partial credits will be given in the marking. If you do not know the answer, give your best guess. You have 3 hours to complete the exam.

1 Robot Move [10 points]

A robot moves on an $n \times m$ rectangle grid. We use the row and column to define the coordinate of a grid location. For instance, $(i, j)$ indicates that the robot is at row $i$ and column $j$. The start point of the robot is at the upper-left corner whose coordinate is $(1, 1)$ and its destination is the right-bottom corner $(n, m)$. The robot can only go one step down or one step to the right in each move. We want to know how many paths the robot can take to reach the target. To make the question more interesting, we set some blocks on the grid: Let $A$ be an $n \times m$ array, the robot can move to $(i, j)$ if $A(i, j) = 0$, and cannot if $A(i, j) = 1$. Design your algorithm to solve the problem and write a function using pseudo code. Analyze the complexity of your algorithm in the $O$ notation.

2 3Sum [10 points]

Design a $O(n^2)$ algorithm to find 3 numbers from an integer array of length $n$ so that their sum equals 0. [Hint: use the $x + y = s$ algorithm as we discussed in the class as a subroutine].

3 Closest Number [10 points]

Given a sorted integer array and an input $x$, find the closest number in the array to $x$. Write pseudo code. Analyze the complexity of your algorithm.

4 Special MST [10 points]

Suppose all edge weights in a graph $G$ are equal. Give an algorithm to compute an MST.

5 Find Bottleneck Edges [10 points]

In a directed graph, given a start node and an end node, design an algorithm to find the minimum number of edges that need to be removed so that the start node and end node are disconnected. [Hint: consider using maxflow-mincut algorithm].
6 Downtown Skyline [10 points]

Given a sequence of intervals that indicate the start and end points of each building. Note that these intervals may overlap. We also have the height of each building. Construct an algorithm to compute the skyline of the group of buildings.

![Figure 1: A skyline of a city.](image)

7 Find String in 2D Pattern [10 points]

Given a 2D array that contains letters, find a string in the array. If the string is not in the pattern, report it. The successive letters in the string should be neighbors in the array. Here we define the neighbors as the letters having row difference of one or column difference of one. For example, CSBC is in the following 2D array:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>G</td>
<td>S</td>
<td>B</td>
<td>L</td>
</tr>
<tr>
<td>H</td>
<td>J</td>
<td>Q</td>
<td>C</td>
<td>Y</td>
</tr>
<tr>
<td>I</td>
<td>R</td>
<td>M</td>
<td>O</td>
<td>K</td>
</tr>
</tbody>
</table>

8 Packing Slabs [10 points]

A slab is a 2 × 1 rectangle patch, how many different ways can you pack a 2 × n rectangle region with n slabs. Design an algorithm to solve the problem. Analyze the complexity of your algorithm.

![Figure 2: A 2 × 10 rectangle filled with 10 slabs.](image)

9 Longest Palindrome Prefix [10 points]

A palindrome is a string that reads the same from left to right or from right to left. Design an efficient algorithm to find the longest palindrome prefix of a string. For example, the longest palindrome prefix for RADARDETECTOR is RADAR and the longest palindrome prefix for BOSTON is B.