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Bayes networks specification and reasoning example

A Bayes network is specified by providing the following information:

- A directed graph that has the network's variables as its nodes, and that has directed arcs into each node, n, from all nodes that directly influence that node. The influencing nodes are the *parents* of node n. The node in question is asssumed to be conditionally independent of all non-descendant nodes given its immediate parents in the network.
- Prior probabilities for all nodes that have no parents.
- For each node n with parents $m_1, \cdots m_k$, a table of all conditional probabilities of the form $P(n \mid m_1 = v_1, \cdots m_k = v_k)$, where each v_i is a possible value of the corresponding variable (node) m_i.

Fig. 1 shows the structure of a small Bayes network. A complete specification of this network would include the graph shown, plus prior probabilities for the "founder" nodes A, B, and C, and a conditional probability table for each of D and E given its respective parents. If all variables were binary (two possible values), and assuming that no shortcuts are taken by exploiting the constraint that probabilities sum to 1 when considering all possible conditioning scenarios for a given variable, this would require 2 prior probabilities for each of the founder nodes (6 values), plus 8 conditional probabilities for the two values of D given all possible combinations of values of its two parents, plus 4 conditional probabilities for the two values of E given the possible values of its sole parent, for a total of 18 values. Contrast this total with the number of values that would be needed to specify the joint distribution of the five network variables in the absence of any conditional independence assumptions (32).

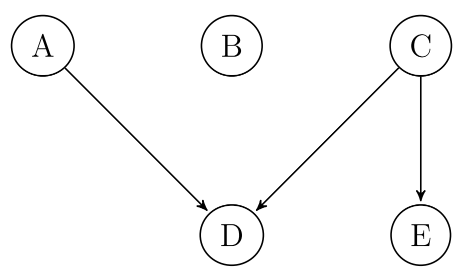

Figure 1: Small Bayes network.

As an example of diagnostic reasoning in ths network, consider the problem of computing the probability that C is **True**, assuming that D is observed to be **True**. The network specification does not provide this probability directly. Instead, Bayes rule is used to manipulate the known "forward" probabilities in order to obtain the desired "backward" probability:

$$P(C \mid D) = \frac{P(C)P(D \mid C)}{P(D)}$$

If D had C as its only parent, this would solve the problem, as the probability $P(D \mid C)$ is provided in the network specification. However, D has an additional parent, A, that is not named on the right-hand side of the above expression. Therefore, it is necessary to split the event that D and C are **True** into the disjoint union of two events in which the state of the other parent is known also:

$$C \cap D = (A \cap C \cap D) \bigcup ((\neg A) \cap C \cap D)$$

It follows that the desired conditional probability may be expressed as follows:

$$P(C \mid D) = \frac{P(A)P(C)P(D \mid A, C) + P(\neg A)P(C)P(D \mid \neg A, C)}{P(D)}$$

The probability of D in the denominator must be expanded as a sum over the four possible combinations of values of D's parents, A and C, but that is an easy "forward" reasoning task. Once that has been done, all needed probabilities in the resulting expression for $P(C \mid D)$ will be available in the network specification.